MATHEMATICAL METHODS
IN
SCATTERING THEORY
AND
BIOMEDICAL ENGINEERING
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Proceedings of the Seventh International Workshop

MATHEMATICAL METHODS
IN
SCATTERING THEORY
AND
BIOMEDICAL ENGINEERING

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Editors

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World Scientific
Preface

This volume of proceedings consists of the papers presented during the 7th International Workshop on Mathematical Methods on Scattering Theory and Biomedical Engineering, held in Nikios School, in Nymphaio, Western Macedonia, Greece, on 8–11 September 2005. The Workshop is organized every two years by the University of Ioannina, the University of Patras, the National Technical University of Athens and the Institute of Chemical Engineering and High Temperature Chemical Process. This year, in the event, the University of Western Macedonia acted on the host institute and supported the organisation of the Workshop.

Once again, the aims of the workshop of bringing together people who have an interest in, or are carrying out research in the even increasing areas of scattering theory and biomedical engineering, have been achieved. The papers and discussions put forward have all been of a high standard and we believe that we had an exciting workshop. It is clear from the interest in the Workshop from researchers across Europe and the World that research is progressing in scattering and biomedical engineering. We hope that through sharing ideas, solutions, and work in progress, we can speed up the potential benefits of cooperation. One of the main benefits to those attending the workshop is the informal discussions that we cannot report in this volume. We are sure that through these discussions further research, research networks and research projects will be generated.

The 7th Workshop was dedicated to Prof. Stylianos Orphanoudakis who passed away in March 2005. Prof. Orphanoudakis was the chairman of the Board of Directors of the Foundation of Research and Technology Hellas (FORTH) and one of the leading personalities in the field of Biomedical Informatics and Engineering in Greece and worldwide.

The Workshop Organizing Committee takes the opportunity to thank all the authors for their contributions. We thank also the University of Ioannina, the National Technical University of Athens, the University of Western Macedonia, the Ministry of Education, the Municipality of Nymphaio, the Municipality of Kozani and Mr. Theodoros Kolokas for their financial support. We are thankful also to Mrs. Evi Kalabakioti and Ms. Lamprina Dimolika for the excellent organization of the Workshop.

Ioannina, January 2006

Dimitrios I. Fotiadis and Christos V. Massalas
University of Ioannina
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Contents

Preface v

Scattering Theory 3

A Method to Solve Inverse Scattering Problems for Electromagnetic Fields in Chiral Media 3
   C. Athanasiadis and E. Kardasi

Electromagnetic Scattering by a Metallic Spheroid 11
   A.D. Kotsis and J.A. Roumeliotis

A New Linear Sampling Method for the Electromagnetic Imagining of Buried Objects 19
   F. Cakoni and H. Haddar

The Energy Functionals for Anisotropic Scattering 31
   G. Dassios and K.S. Karadima

Nonlinear Integral Equations in Inverse Obstacle Scattering 39
   O. Ivanyshyn and R. Kress

3D Wave Scattering by Acoustic Screens and Cracks Embedded in Acoustic and Elastic Media, Making Use of the Traction Boundary Element Method 51
   A. Tadeu, P. Amado Mendes and J. António

Point Source Electromagnetic Excitation of a Layered Sphere 63
   N.L. Tsitsas and C. Athanasiadis

Scattering by an Infinite Elliptic Metallic Cylinder 71
   G.D. Tsogkas, J.A. Roumeliotis and S.P. Savaidis

Applied Mathematics 81

Chaotic Dynamics Applied on Time-Prediction of Earthquakes 83
   I. Arahovitis
Homogenization in Chiral Elasticity
G. Barbatis and I.G. Stratis

Target Identification of Partially Coated Objects Using Electromagnetic Waves
D.L. Colton

Discretization – Optimization Methods for Relaxed Optimal Control Problems
I. Chryssoverghi, I. Coletsos and B. Kokkinis

The Factorization Method for an Acoustic Wave Guide
A. Charalambopoulos, D. Gintides, K. Kiriaki and A. Kirsch

General Polarizability Tensor For Two Spheres
G. Dassios, M. Hadjinicolaou and G. Kamvyssas

Spheroidal Semiseparation in Stokes Flow Revisited
G. Dassios and P. Vafeas

Shape Control and Damage Identification of Piezoelectric Smart Beams Using Finite Element Modelling and Genetic Optimization
E.P. Hadjigeorgiou, C.V. Massalas and G.E. Stavroulakis

On Generalized Linear Matrix Difference Systems
G.I. Kalogeropoulos, D.P. Papachristopoulos and S.C. Giotopoulos

Neural Network Classification of Acoustic Emission and Drop Signals
V. Kappatos and E. Dermatas

A BEM solution of the Boussinesque Problem in Solids with Microstructure
G.F. Karlis, S.V. Tsinopoulos and D. Polyzos

Stochastic Differential Equations of Sobolev Type in Infinite Dimensional Hilbert Spaces
K.B. Liaskos, I.G. Stratis and A.N. Yannacopoulos
A Nonlocal Formulation of the DBAR Formalism and Boundary Value Problems in Two Dimensions

D.A. Pinotsis

A Fast Numerical Method for a Simplified Phase Field Model

C.A. Sfyrrakis and V.A. Dougalis

Multiple Solutions for Nonlinear Hemivariational Inequalities Below the First Eigenvalue

G. Smyrlis and D. Kravvaritis

Mode-I Crack Profile in Materials with Microstructural Effects: A Numerical Solution

K.G. Tsepoura, S.V. Tsinopoulos and D. Polyzos

Analytic Inversion of Matrices with 2k×2k Circulant Blocks

N.L. Tsitsas, E.G. Alivizatos and G.H. Kalogeropoulos

A Numerical Study on the Propagation of Transient Elastic Waves in Axisymmetric Vessels

V. Vavourakis and D. Polyzos

Biomedical Engineering

Biocomplexity of Respiratory Neural Network During Eupnea Gasping and Hypercapnia

M. Akay

Tinnitus Diagnosis and Therapy Method Employing Ultrasound Dithering

A. Czyzewski and J. Klejza

On the Hidden Electromagnetic Activity of the Brain

G. Dassios

Analysis of EEG Images

G. Dassios, S.N. Giapalaki, A.N. Kandili and F. Kariotou
A Decision Tree Based Approach for the Identification of Ischaemic Beats in ECG Recordings

T.P. Exarchos, C. Papaloukas and D.I. Fotiadis

Nonlinear Models of Artery Dynamics

P. Kalita, M. Paszynski and R. Schaefer

Audiovisual Speech Recognition for Training Hearing Impaired Patients

B. Kostek, P. Dalka and A. Czyzewski

A Detailed Mathematical Model of Diffused Brain Edema Early Detection

V. Kostopoulos, C. Derdas and E. Douzinas

Nonlinear Physiological Systems Identification: Application to Cerebral Hemodynamics Under Orthostatic Stress

G.D. Mitsis, R. Zhang, B.D. Levine and V.Z. Marmarelis

Detecting and Localizing True Brain Interactions from EEG/MEG Data

G. Nolte

An Automatic Microcalcification Detection System Utilizing Mammographic Enhancement Techniques

A.N. Papadopoulos and D.I. Fotiadis

Feature Extraction from Interictal Epileptic and Non-Epileptic Pathological EEG Events for Diagnostic Purposes using LVQ1 Neural Network

S. Papavlasopoulos, M. Poulos and A. Evangelou

Immune System – Based Clustering and Classification Algorithms

D.N. Sotiropoulos and G.A. Tsihrintzis

Clustering and Classification of Electrophoresis Strands for Fungi Fingerprinting

I.O. Stathopoulou, G.A. Tsihrintzis, K. Kollia and A. Velegraki

Multidimensional Cardiac Models

D.G. Tsalikakis, G.P. Kremmydas and D.I. Fotiadis
A Framework for Fuzzy Expert System Creation

M. Tsipouras, C.A. Voglis, I.E. Lagaris and D.I. Fotiadis

423

Mobile and Electronic Medical Support and Education for Dyslexic Students

M. Virvou and E. Alepis

431

Author Index

439
Scattering Theory
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A METHOD TO SOLVE INVERSE SCATTERING PROBLEMS FOR ELECTROMAGNETIC FIELDS IN CHIRAL MEDIA

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A method is proposed to study inverse scattering problems for electromagnetic fields in chiral media. The direct scattering problem for the perfect conductor and the dielectric is formulated in its dyadic form considering that the space in the exterior of the chiral scatterer is also an infinite chiral medium in order to present the problem in its general form. Herglotz dyadic fields in chiral media are defined. Superposition theorems are proved, far-field operators are defined and integral equations are posed. The inversion scheme is based on the unboundedness of the solution of the integral equation and on the properties of Herglotz dyadics.

1. Introduction

In the present work we develop an approximation method for the inverse obstacle scattering problem in chiral media based on Herglotz dyadics. Scattering theorems for dyadic electromagnetic fields in chiral media have been proved in [3]. Herglotz functions in chiral media (Beltrami Herglotz functions and chiral Herglotz pairs) have been defined and studied for the vector case in [4]. In [6] Beltrami Herglotz dyadics and dyadic electromagnetic Herglotz pairs have been defined for the dyadic case. The method we develop forms an extension of the Colton and Kirsch method for acoustics (see [8,9]). In [11] Colton and Kress study inverse problems in acoustics and electromagnetics. Dassios and Rigou in [13] and Gintides and Kiriaki in [14] study inverse problems in elasticity. We develop an inverse scattering method for electromagnetic fields in chiral media when the scatterer is a perfect conductor or a dielectric.

Thus, in Section 2 we formulate the direct perfect conductor problem and the transmission problem for electromagnetic dyadic fields in chiral...
media using Bohren decomposition of electromagnetic fields into suitable dyadic Beltrami fields. We define the LCP and the RCP Beltrami Herglotz dyadics and the dyadic electromagnetic Herglotz pairs. In Section 3 we prove a superposition theorem when the scatterer is a perfect conductor or a dielectric. Far field operators are defined and an integral equation is posed. Finally, an inversion scheme is posed and a theorem for its solvability is proved.

2. Dyadic formulation in chiral media

We consider a time-harmonic plane dyadic electromagnetic wave \( (\vec{E}, \vec{H}) \) propagating in a homogeneous isotropic chiral medium \( \Omega \) with electric permittivity \( \varepsilon \), magnetic permeability \( \mu \) and chirality measure \( \beta \). Let \( \Omega^- \) be a bounded and closed subset of \( \mathbb{R}^3 \) with \( C^2 \)-boundary \( S = \partial \Omega^- \), filled with a homogeneous isotropic chiral medium with corresponding physical parameters \( \varepsilon^-, \mu^- \) and \( \beta^- \). The set \( \Omega^- \) will be referred to as the scatterer and it will be considered to be either a perfect conductor or a dielectric.

During its propagation, the electromagnetic field \( (\vec{E}, \vec{H}) \) is incident upon the scatterer \( \Omega^- \) and the scattered field \( (\vec{E}^s, \vec{H}^s)\) is produced. Then, the total electromagnetic field \( (\vec{E}, \vec{H}) \) in \( \Omega \) is given by

\[
\vec{E}(r) = \vec{E}^i(r) + \vec{E}^s(r) \quad \text{and} \quad \vec{H}(r) = \vec{H}^i(r) + \vec{H}^s(r) \quad \text{in} \quad \Omega. \tag{1}
\]

An electromagnetic field \( (\vec{E}, \vec{H}) \) in \( \Omega \) solves the equations (see [3,6])

\[
\nabla \times \vec{E}(r) = \beta \gamma^2 \vec{E}(r) + i \omega \mu \left( \frac{\gamma}{\kappa} \right)^2 \vec{H}(r) \quad \text{in} \quad \Omega, \tag{2}
\]

\[
\nabla \times \vec{H}(r) = \beta \gamma^2 \vec{H}(r) - i \omega \varepsilon \left( \frac{\gamma}{\kappa} \right)^2 \vec{E}(r) \quad \text{in} \quad \Omega, \tag{3}
\]

where \( \omega \) is the angular frequency and \( \kappa^2 = \omega^2 \varepsilon \mu \), \( \gamma^2 = \kappa^2 (1 - \beta^2 \kappa^2)^{-1} \). The total interior electromagnetic field in \( \Omega^- \) satisfies also (2) and (3) with physical parameters \( \varepsilon^-, \mu^- \) and \( \beta^- \).

The scattered field is assumed to satisfy the equivalent Silver–Müller radiation conditions

\[
\vec{E}^s(r) + \eta \hat{r} \times \vec{H}^s(r) = o \left( \frac{1}{r} \right) \quad \text{and} \quad \hat{r} \times \vec{E}^s(r) - \eta \vec{H}^s(r) = o \left( \frac{1}{r} \right) \quad r \to \infty, \tag{4}
\]

uniformly in all directions \( \hat{r} = r/r \), with \( r = |r| \), where \( \eta = (\mu/\varepsilon)^{1/2} \) is the intrinsic impedance of the chiral medium in \( \Omega \).
In a homogeneous isotropic chiral medium the electromagnetic fields are composed of Left-Circularly Polarized (LCP) and Right-Circularly Polarized (RCP) components, which have different wavenumbers and independent directions of propagation. We consider the Bohren decomposition of \( \vec{E} \) and \( \vec{H} \), [3], [15]
\[
\vec{E}(r) = \vec{E}_L(r) + \vec{E}_R(r), \quad \vec{H}(r) = \frac{1}{i\eta}(\vec{E}_L(r) - \vec{E}_R(r)).
\] (5)
The LCP and RCP Beltrami fields \( \vec{E}_L \) and \( \vec{E}_R \) respectively, satisfy the Beltrami equations
\[
\nabla \times \vec{E}_L = \gamma_L \vec{E}_L, \quad \nabla \times \vec{E}_R = -\gamma_R \vec{E}_R,
\] (6)where the wavenumbers \( \gamma_L \) and \( \gamma_R \) are given by
\[
\gamma_L = \frac{\kappa}{1 - \kappa\beta}, \quad \gamma_R = \frac{\kappa}{1 + \kappa\beta},
\] they are divergence free and also satisfy the dyadic Helmholtz equation
\[
\Delta \vec{E}_A + \gamma_A^2 \vec{E}_A = \vec{O},
\] (7)for \( A=L,R \).
Using, now, the Silver-Müller radiation conditions (4) for scattered electromagnetic waves and the relations (6) we derive the following radiation conditions, [3],
\[
\hat{r} \times \vec{E}_L^s + i\vec{E}_L^t = o\left(\frac{1}{r}\right), \quad \hat{r} \times \vec{E}_R^s - i\vec{E}_R^t = o\left(\frac{1}{r}\right), \quad r \to \infty,
\] (8)uniformly for all directions \( \hat{r} \).

For a perfect conductor the total Beltrami fields \( \vec{E}_L^t, \vec{E}_R^t \) satisfy the boundary condition
\[
\hat{n} \times \vec{E}_L^t = -\hat{n} \times \vec{E}_R^t \quad \text{on} \quad S.
\] (9)
For a dielectric the total exterior and interior Beltrami fields \( \vec{E}_L^t, \vec{E}_R^t \) and \( \vec{E}_L^- , \vec{E}_R^- \) respectively satisfy the transmission conditions
\[
\hat{n} \times (\vec{E}_L^t - \vec{E}_L^-)(r) = \hat{n} \times (\vec{E}_R^- - \vec{E}_R^t)(r) \quad \text{on} \quad S,
\] (10)
\[
\eta^- \hat{n} \times (\vec{E}_L^t - \vec{E}_L^-)(r) = \eta^t \hat{n} \times (\vec{E}_R^t - \vec{E}_R^-)(r) \quad \text{on} \quad S.
\] (11)
The solvability of the perfect conductor and the transmission problem has been studied in [1] and [2] respectively, where existence and uniqueness of solution has been proved.
If, now, the unit vectors $\hat{d}_L$ and $\hat{d}_R$ describe the directions of propagation of the LCP and RCP wave, respectively, then the incident plane dyadic electric field assumes the form

$$\tilde{E}_i(r|\hat{d}_L, \hat{d}_R) = \tilde{E}_L^i(r|\hat{d}_L) + \tilde{E}_R^i(r|\hat{d}_R), \quad (12)$$

where

$$\tilde{E}_L^i(r|\hat{d}_L) = K_L(\hat{d}_L)e^{i\gamma_L \hat{d}_L \cdot r}, \quad \tilde{E}_R^i(r|\hat{d}_R) = K_R(\hat{d}_R)e^{i\gamma_R \hat{d}_R \cdot r}, \quad (13)$$

are the LCP and RCP plane dyadic electric fields respectively. The dyadics $K_L$ and $K_R$ are given by, [3]

$$K_L(\hat{d}_L) = \hat{I} - \hat{d}_L \hat{d}_L + i\hat{d}_L \times \hat{I}, \quad K_R(\hat{d}_R) = \hat{I} - \hat{d}_R \hat{d}_R - i\hat{d}_R \times \hat{I}, \quad (14)$$

where $\hat{I}$ is the identity dyadic.

Finally, we define Herglotz dyadics in chiral media, [6]. A Beltrami Herglotz dyadic is a dyadic field of the form

$$\tilde{E}_A(r) = q_{A1}(r) \otimes \hat{e}_1 + q_{A2}(r) \otimes \hat{e}_2 + q_{A3}(r) \otimes \hat{e}_3, \quad (15)$$

where $q_{Ai}, A = L, R, i = 1, 2, 3$ are three Beltrami Herglotz vector functions and $\{\hat{e}_1, \hat{e}_2, \hat{e}_3\}$ denotes the orthonormal base in $\mathbb{R}^3$. A Beltrami Herglotz dyadic satisfies the well known Herglotz condition and has the following representation form

$$\tilde{E}_A(r) = \int_{S^2} \tilde{b}_A(\hat{d}_A)e^{i\gamma_A \hat{d}_A \cdot r}d\hat{d}_A, \quad A = L, R, \quad (16)$$

where $\tilde{b}_A \in \tilde{T}_A^2(S^2), A = L, R$ and

$$\tilde{T}_L^2(S^2) = \{\tilde{b}_L: S^2 \rightarrow \mathbb{C}^9: \tilde{b}_L \in L^2(S^2), \hat{n} \cdot \tilde{b}_L = 0, \hat{n} \times \tilde{b}_L = -i\tilde{b}_L\}, \quad (17)$$
$$\tilde{T}_R^2(S^2) = \{\tilde{b}_R: S^2 \rightarrow \mathbb{C}^9: \tilde{b}_R \in L^2(S^2), \hat{n} \cdot \tilde{b}_R = 0, \hat{n} \times \tilde{b}_R = i\tilde{b}_R\}. \quad (18)$$

A dyadic electromagnetic Herglotz pair is defined by

$$\tilde{E}(r) = \tilde{E}_L(r) + \tilde{E}_R(r), \quad \tilde{H}(r) = \frac{1}{i\eta}(\tilde{E}_L(r) - \tilde{E}_R(r)), \quad (19)$$

and represents entire solution to equations (2) and (3).

We call the dyadic field $\tilde{b} = \tilde{b}_L + \tilde{b}_R$ for $\tilde{b}_A \in \tilde{T}_A^2(S^2), A = L, R$, the electric Herglotz kernel for the electric Herglotz dyadic $\tilde{E}$ and we denote
the set of all electric Herglotz kernels for the electric Herglotz dyadic \( \tilde{E} \) by \( \tilde{T}^2_{LR}(S^2) \). In [6] it has been proved that the set of dyadic electromagnetic Herglotz pairs is dense within the set of the solutions of equations (2), (3), that is, for every solution \( \tilde{E} \) and \( \tilde{H} \) of equations (2), (3) and for every \( \epsilon > 0 \) there exists a dyadic electromagnetic Herglotz pair \( (\tilde{E}, \tilde{H}) \), such that

\[
\max_{r \in \Omega^+} \| \tilde{E}(r) - \tilde{E}(r') \| \leq \epsilon, \quad \max_{r \in \Omega^+} \| \tilde{H}(r) - \tilde{H}(r') \| \leq \epsilon. \tag{20}
\]

3. An inverse scattering method

Now that we have defined Herglotz dyadics we will make use of them to prove a superposition theorem for electromagnetic dyadic fields in chiral media when the scatterer is a perfect conductor or a dielectric.

**Theorem 3.1.** Given dyadic densities \( \tilde{b}_A \in \tilde{T}^2_A(S^2) \), \( A = L, R \), when the incident electric field is of the form

\[
\tilde{E}^i(r|\hat{d}_L, \hat{d}_R) = \int_{S^2} \tilde{b}_L(\hat{d}_L)e^{i\gamma_L d_L \cdot r} ds(\hat{d}_L) + \int_{S^2} \tilde{b}_R(\hat{d}_R)e^{i\gamma_R d_R \cdot r} ds(\hat{d}_R), \tag{21}
\]

then, the scattered field is given by the formula

\[
\tilde{E}^s(r|\hat{d}_L, \hat{d}_R) = \int_{S^2} \tilde{b}_L(\hat{d}_L) \cdot \{ \tilde{E}^s_L(r|\hat{d}_L) + \tilde{E}^s_R(r|\hat{d}_L) \} ds(\hat{d}_L) \\
+ \int_{S^2} \tilde{b}_R(\hat{d}_R) \cdot \{ \tilde{E}^s_L(r|\hat{d}_R) + \tilde{E}^s_R(r|\hat{d}_R) \} ds(\hat{d}_R), \tag{22}
\]

and has the far-field pattern

\[
\tilde{E}^\infty(r|\hat{d}_L, \hat{d}_R) = \int_{S^2} \tilde{b}_L(\hat{d}_L) \cdot \{ \tilde{E}^\infty_L(r|\hat{d}_L) + \tilde{E}^\infty_R(r|\hat{d}_L) \} ds(\hat{d}_L) \\
+ \int_{S^2} \tilde{b}_R(\hat{d}_R) \cdot \{ \tilde{E}^\infty_L(r|\hat{d}_R) + \tilde{E}^\infty_R(r|\hat{d}_R) \} ds(\hat{d}_R), \tag{23}
\]

where \( \tilde{E}^\infty_A, A = L, R \) are the corresponding to \( \tilde{E}_A^s \) far-field patterns.

**Proof.** We use the dyadic potentials

\[
[S^-(k)\tilde{\alpha}](r) = \int_{S^2} \tilde{\alpha}(r') \Phi(r, r'; k) ds(r'), \quad r \in \Omega^-, \tag{24}
\]

\[
[C^-(k)\tilde{\alpha}](r) = \nabla \times [S^-(k)\tilde{\alpha}](r), \quad [F^-(k)\tilde{\alpha}](r) = \nabla \times [C^-(k)\tilde{\alpha}](r), r \in \Omega^-, \tag{25}
\]

where \( \Phi(r, r'; k) \) is the fundamental solution of the Helmholtz equation \( \Delta u + ku^2 = 0 \) which is given by \( \Phi(r, r'; k) = \frac{e^{ik|r-r'|}}{4\pi|r-r'|} \), \( r \neq r' \).